MODULE 5
KINEMATICS

INTRODUCTION TO DYNAMICS

Dynamics is the branch of science which deals with the study of behaviour of body or particle in the state of motion under the action of force system. The first significant contribution to dynamics was made by Galileo in 1564. Later, Newton formulated the fundamental laws of motion.

Dynamics branches into two streams called kinematics and kinetics.

Kinematics is the study of relationship between displacement, velocity, acceleration and time of the given motion without considering the forces that causes the motion, or Kinematics is the branch of dynamics which deals with the study of properties of motion of the body or particle under the system of forces without considering the effect of forces.

Kinetics is the study of the relationships between the forces acting on the body, the mass of the body and the motion of body, or Kinetics is the branch of dynamics which deals with the study of properties of motion of the body or particle in such way that the forces which cause the motion of body are mainly taken into consideration.

TECHNICAL TERMS RELATED TO MOTION

Motion: A body is said to be in motion if it is changing its position with respect to a reference point.

Path: It is the imaginary line connecting the position of a body or particle that has been occupied at different instances over a period of time. This path traced by a body or particle can be a straight line/liner or curvilinear.

Displacement and Distance Travelled

Displacement is a vector quantity, measure of the interval between two locations or two points, measured along the shortest path connecting them. Displacement can be positive or negative.

Distance is a scalar quantity, measure of the interval between two locations measured along the actual path connecting them. Distance is an absolute quantity and always positive.

A particle P is moving along the x axis. Let O be the origin. The position vector OP can be defined this position P by the expression OP = x1i + x2j + x3k, if the origin is at the point O (Figure 5.1)

Let,
P —> Position of the particle at any time t 1
x₁—> Displacement of particle measured in +ve direction of O
x₂—> Displacement of particle measured in -ve direction of O

In this case the total distance travelled by a particle from point O to P to P₁ and back to O is not equal to displacement.

Total distance travelled = x₁+ x₁ + x₂ + x₂ = 2(x₁ + x₂).
Whereas the net displacement is zero.

**Velocity:** Rate of change of displacement with respect to time is called velocity denoted by v.

Mathematically v = dx/dt

**Average velocity:** When an object undergoes change in velocities at different instances, the average velocity is given by the sum of the velocities at different instances divided by the number of instances. That is, if an object has different velocities v₁, v₂, v₃, ..., vₙ, at times t = t₁, t₂, t₃, ..., tₙ, then the average velocity is given by

\[ V = \frac{(v₁+v₂+v₃+...+vₙ)}{n} \]

**Instantaneous velocity:** It is the velocity of moving particle at a certain instant of time. To calculate the instantaneous velocity Δx is considered as very small.

Instantaneous velocity \[ v = \Delta t \to 0 \frac{\Delta x}{\Delta t} \]

**Speed:** Rate of change of distance travelled by the particle with respect to time is called speed.

**Acceleration:** Rate of change of velocity with respect to time is called acceleration

Mathematically a = dv/dt

**Average Acceleration**

Consider a particle P situated at a distances of x from O at any instant of time t having a velocity v. Let P₁ be the new position of particle at a distance of (x + Δx) from origin with a velocity of (v + Δv). See Figure 5.2.
Average acceleration over a time \( t \), is given by
\[
a_{\text{avg}} = \frac{\Delta v}{\Delta t}
\]

**Acceleration due to gravity**: Each and everybody is attracted towards the centre of the earth by a gravitational force and the acceleration with which the body is pulled towards the centre of the earth due to gravity is denoted by 'g'. The value of g is normally taken as 9.81 m/s\(^2\).

**Newton's Laws of Motion**

**Newton's first law**: This law states that 'everybody continues in its state of rest or of uniform motion, so long as it is under the influence of a balanced force system'.

**Newton's second law**: This law states that 'the rate of change momentum of a body is directly proportional to the impressed force and it takes place in the direction of force acting on it.'

**Newton's third law**: This law states that 'action and reaction are equal in magnitude but opposite in direction'.

**Types of Motion**
1. Rectilinear motion
2. Curvilinear motion
3. Projectile motion

**Graphical representation**: The problems in dynamics can be analysed both analytically and graphically without compromising on the accuracy. Most of the times graphical representations can lead to simpler solutions to complicated problems. Using the simple terms defined in the initial portions of the section, we can draw different types of graphs.

**Displacement-time graph**: The representation with graph in Figure 5.3 shows that the displacement is uniform with time. Hence it is understood that the body is under rest as the displacement is constant with respect to time.

The representation with graph in Figure 5.4 shows that the plot is having a constant slope and the variation of displacement is uniform with time. The slope indicates the ratio of displacement to time which is equal to velocity of the body; Hence it is understood that the body is moving under uniform velocity.

Figure 5.5 shows variation of displacement with time as a curve. The tangent to this curve at any point indicates the velocity of the body at that instant. As can be seen the slope of the tangent is changing with respect to time and ever increasing, it indicates that the velocity is changing with respect to time and also indicates that the velocity is increasing with respect to time. This increasing velocity with respect to time is termed acceleration.
In case of Figure 5.6, the curvature is decreasing, and the slope of the tangent is decreasing with respect to time and rate change of velocity is decreasing. This is termed as deceleration.
Figure 5.6  Variation of displacement with time.

**Velocity-time graph:** A plot of velocity with respect to time is termed as velocity-time graph

Figure 5.7  Variation of velocity with time.

Unit of velocity = \( v = LT^{-1} \)
Unit of time = \( T \)
Velocity x Time = \( LT^{-1} \times T = L \rightarrow \) Distance

Hence, the area under V-T graph will produce the distance traveled by the body/particle from time \( t_1 \) to \( t_2 \),

\[ s = v \times (t_2 - t_1) = vt \]  \[ \text{(i)} \]

This is applicable only when the velocity is uniform.

In case of Figure 5.8, the velocity is varying uniformly with respect to time as seen from sloped straight line.
Figure 5.8  Variation of velocity with time.

The slope of the line is gives acceleration

\[ a = \frac{(v_2-v_1)}{(t_2-t_1)} \]
\[ (v_2-v_1) = a(t_2-t_1) \]
\[ v_2 = v_1 + a(t_2-t_1) \]
\[ = u + at \]

where \( v \) = final velocity, \( u \) = initial velocity and \( t = (t_2-t_1) \)

As seen from earlier graph, the total distance traveled is given by the area under the curve and hence the area is given as

\[ S = v_1 \times t + 0.5(v_2-v_1)t \]

**But acceleration = \( \frac{a = (v_2-v_1)}{t} \)**

Substituting, we get

\[ S = v_1 \times t + 0.5 \times at^2 \text{ or } ut + 0.5at^2 \]

where \( u \) is the initial velocity or velocity at time \( t_1 \)

**Acceleration-time graph:*** It is a plot of acceleration versus time graph as shown in Figure 5.9

It is seen that die acceleration is constant with respect to time \( t \). The same can be connected to velocity-time graph (Figure 5.6), wherein the velocity variation is constant. The coordinates in acceleration-time graph show the area under the velocity-time curve.

In case of Figure 5.10, it is seen that the acceleration line in acceleration-time plot, it shows the variation of acceleration to be uniform.
The curve in velocity-time graph will be simplified as a straight line in acceleration-time graph. Using Eqs (1) and (2), to get an equation without time, we substitute for \( t \) from Eq. 1 in Eq. 2, we get

\[
S = \frac{(v-u)}{a} + 0.5\left[\frac{(v-u)}{a}\right]^2
\]

\[
v^2 - u^2 = 2as
\]

\[
\ldots (3)
\]

**Rectilinear Motion**
When a particle or a body moves along a straight line path, then it is called linear motion or rectilinear motion.

Equation of motion along a straight line
\[ v = u + at \]
\[ v^2 - u^2 = 2as \]
\[ s = ut + 0.5at \]

Example 1: The motion of a particle is given by the equation \[ x = t^3 - 3t^2 - 9t + 12. \] Determine the time, distance travelled and acceleration of particle when velocity becomes zero.

Solution
\[ X = t^3 - 3t^2 - 9t + 12 \] (1)

Differentiating Eq. (1) with respect to 'x', we get
\[ v = \frac{dx}{dt} = 3t^2 - 6t - 9 \] (2)
when \( v = 0 \)
The above equation is in the form of and the solution is

\[ ax^2 + bx + c = 0 \]

\[ x = -b \pm \sqrt{b^2 - 4ac} / 2a \] (3)
substituting the respective values in Eq. (3), we get

\[ t = -1 \text{ or } t = 3 \text{ s} \] (negative value of t can be discarded)
Substitute \( t = 3 \text{ s} \) in (1), we get
\[ x = -15 \text{ m} \]
Differentiating Eq. (2), we get
\[ a = 12 \text{ m/s}^2 \]

Example 2: The motion of a particle is defined by the relation \( x = t^3 - 9t^2 + 24t - 6. \) Determine the position, velocity and acceleration when \( t = 5 \text{ s}. \)

Solution
\[ x = t^3 - 9t^2 + 24t - 6 \] (1)

Differentiating Eq. (1), we get
\[ \frac{dx}{dt} = v = 3t^2 - 18t + 2 \] (2)

Differentiating Eq. (2), we get
\[ \frac{d^2x}{dt^2} = a = 6t - 18 \]

Substitute \( t = 5 \text{ s} \) in Eqs. (1), (2) and (3), we get
\[ x = 14 \text{ m} \]
\[ v = 9 \text{ m/s} \]
\[ a = 12 \text{ m/s}^2 \]
Elements of Civil Engineering and Engineering Mechanics

Example 3: A car is moving with a velocity of 15 m/s. The car is brought to rest by applying brakes in 5 s. Determine (i) Retardation (ii) Distance travelled by the car after applying the brakes.

Solution
(i) Retardation
We know that 
\[ v = u + at \]
\[ 0 = 15 + ax5 \]
\[ a = -3 \text{ m/s}^2 \]
(ii) Distance travelled by the car after applying the brakes.
We know that
\[ s = ut + 0.5at^2 \]
\[ s = 15x5 + 0.5x(-3)x(5)^2 \]
\[ s = 37.5 \text{m} \]

MOTION UNDER GRAVITY

We know that everybody on the earth experiences a force of attraction towards the centre of the earth is known as gravity. When a body is allowed to fall freely, it is acted upon by acceleration due to gravity and its velocity goes on increasing until it reaches the ground. The force of attraction of the earth that pulls all bodies towards the centre of earth with uniform acceleration is known as acceleration due to gravity. The value of acceleration due to gravity is constant in general and its value is considered to be 9.81 m/s\(^2\) and is always directed towards the centre of earth. Acceleration due to gravity is generally denoted by 'g'.

When the body is moving vertically downwards, the value of g is considered as positive and if the body is projected vertically upwards, then acceleration due to gravity is considered as negative. Evidently, all equations of motion are applicable except by replacing uniform acceleration V with acceleration due to gravity 'g' and are written as
(i) When a body is projected vertically downward, under the action of gravity, the equations of motion are
\[ v = u + gt \]
\[ v^2 = u^2 + 2gh \]
\[ h = ut + 0.5gt^2 \]
(ii) When a body is projected vertically upward, under the action of gravity, the equations of motion are
\[ v = u - gt \]
\[ v^2 = u^2 - 2gh \]
\[ h = ut - 0.5gt^2 \]

Example 4: A ball is thrown vertically upward into air with an initial velocity of 35 m/s. After 3 s another ball is thrown vertically. What initial velocity must be the second ball has to pass the first ball at 30 m from the ground.

Solution 
Consider the first ball, we know that
\[ h = u_1t - 0.5gt^2 \]
\[ 30 = 35t - 0.5\times9.81\times t^2 \]
\[ t^2 - 7.135t + 6.116 = 0 \]
\[ t = 6.138 \text{s} \]
Consider the second ball

\[ t_2 = (6.138 -3) = 3.138\text{s} \]
\[ h = u_2t_2 - 0.5gt_2^2 \]
\[ h = 30\text{ m} \]
\[ u_2 = 24.91\text{m/sec} \]

**CURVILINEAR MOTION**

**Introduction**

When a moving particle describes a path other than a straight line is said be a particle in curvilinear motion. If the curved path lies in a single plane is called plane curvilinear motion. Most of the motions of particles encountered in engineering practices are of this type.

**Curvilinear Motion in Cartesian Coordinates**

In Cartesian coordinates two axes of reference will be chosen. To define the position of particle at any instant of time we have to choose a reference axis namely \( x \) and \( y \).

Let, \( P \) be the position of particle at any instant of time \( t \)

'\( P_1 \)' be the new position at an instant of time \((t + \Delta t)\) from origin.

Join \( O \) to \( P \) and \( O \) to \( P_1 \)

Let \( r \) be the position vector of \( P \) having magnitude and direction.

\( r_1 \) be the position vector \( P_1 \)

\( \Delta r \) be the rate of change in displacement amount over a time \( \Delta t \)

Average velocity over a time \( \Delta t = \Delta r/\Delta t \)

![Figure 5.11](image-url)

Velocity of particle is vector tangent to the path of particle'
Let, $\Delta x$ be the distance travelled in $x$ direction
$\Delta y$ be the distance travelled in $y$ direction

Velocity in 'x' direction $= v_x = \frac{dx}{dt}$
Velocity in $y$ direction $= v_y = \frac{dv}{dt}$
Resultant velocity $= v = \sqrt{(v_x^2 + v_y^2)}$
Normal and tangential component of acceleration:
Velocity of moving particle is always vector tangential to the path of particle. But acceleration is not tangential to path. But it is convenient to resolve the acceleration along tangential and normal direction.

![Diagram of normal and tangential components of acceleration](image)

Figure 5.11

Tangential acceleration = \( \dot{a} = \frac{dv}{dt} \)
Normal acceleration = \( a_n = \left( \frac{v^2}{\rho} \right) \rho = r \)

Where '\( \rho \)' is the radius of curvature.

From the above expression it is evident that tangential component of acceleration is equal the rate of change of velocity with respect to time. Normal component of acceleration is equal to the square of velocity divided by the radius of curvature.

Example 6: The motion of a particle is described by the following equation \( x = 2(t + 1)^2 \), \( y = 2(t + 1)^{-2} \). Show that path travelled by the particle is rectangular hyperbola. Also find the velocity and acceleration of particle at \( t = 0 \)

Solution
To find the path travelled, we know that
\[
\begin{align*}
x &= 2(t+1)^2 \\
y &= 2(t+1)^{-2}
\end{align*}
\]
Multiplying the two equation
\[xy = 2 \quad [xy = \text{constant}]\]

This represents a rectangular hyperbola
We know \( x = 2(t + 1)^2 \)
Component of velocity in x direction \( v_x = 2 \times 2(t + 1) \)

Component of acceleration in x direction \( a_x = \frac{d^2x}{dt^2} = 2 \times 2 = 4 \text{ m/s}^2 \)
When \( t = 0 \), \( v_x = 4 \text{ m/s}^2 \)
\[a_x = 4 \text{ m/s}^2\]

We know \( y = 2(t + 1)^{-2} \)
Component of velocity in y direction

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\[ v_y = \frac{dy}{dt} = 2(-2)(t+1)^{-3} \]
\[ = -4(t+1)^{-3} \]

Component of acceleration in y direction
\[ a_y = \frac{d^2y}{dt^2} = -12(t+1)^{-4} \]

When \( t = 0 \)

\[ v_y = 4 \text{ m/s} \]
\[ a_y = 12 \text{ m/s} \]

\[ \text{velocity} = v = \sqrt{(v_x^2 + v_y^2)} \]
\[ \tan \Theta = \frac{v_y}{v_x} = -1 \]
\[ \Theta = 45^\circ \]

\[ \text{Acceleration} = a = \sqrt{(a_x^2 + a_y^2)} = 12.65 \text{ m/s}^2 \]
\[ \alpha = 71.6^\circ \]

**CURVILINEAR MOTION IN POLAR COORDINATES**

The curvilinear motion of particle can be expressed in terms of rectangular components and components along the tangent and normal to the path of particle.

In certain problems the position of particle is more conveniently described by its polar coordinates. In that case it is much simpler to resolve the velocity and acceleration of particle into components that are parallel and perpendicular to the position vector \( V \) of the particle. These components are called radial and transverse components.

![Figure 5.12](image)

Consider a collar \( P \) sliding outward along a straight rod \( OA \), which itself is rotating about fixed point \( O \). It is much convenient to define the position of collar at any instant in terms of
distance \( r \) from the point 'O' and angular position '\( \Theta \)' of rod OA with x axis.

Thus polar coordinates of point P these are \((r, \Theta)\).

It can be shown that the radial and transverse components of velocity are \( v_r = r \)
(Radial component directed along position vector \( r \))

Transverse component

\[ v_\theta = r \Theta \]

(Transverse component directed along the normal to the position vector \( r \))

Total velocity \( v = \sqrt{v_r^2 + v_\theta^2} \)

Radial component of acceleration \( a_r = r \cdot (r \Theta) \)

![Figure 5.13](image_url)

Transverse component of acceleration \( a_\theta = r \Theta \) + 2 \( r \Theta \)

Total acceleration \( a = \sqrt{(a_r^2 + a_\theta^2)} \)

The component of velocity and acceleration are related as

\[ a_r = v_r - v_\theta \Theta \]

\[ a_r = v_\theta - v_r \Theta \]

From the above equation it can be seen that '\( a_r \)' is not equal to \( v_r \) and \( a_\theta \) is not equal to \( v_\theta \)

It would be noted that radial component of velocity and acceleration are taken to the positive in the same sense of position vector \( r \).

Transverse components of velocity and acceleration are taken to the positive if pointing towards the increasing value of \( \Theta \).

To understand the physical significance of above results let us assume the following two situations.

(i) If \( r \) is of constant length and \( \Theta \) varies. Then \( r \) reduces to rotation along circular path.

\( r = \text{constant} \)

\( r = r = 0 \)

\( v_\theta = r \Theta \)

\( v_r = r = 0 \)
\[ a_\theta = r \theta \]
\[ a_r = -r(\theta)^2 \]

(-ve sign indicates that \( a_r \) is directed opposite to the sense of position vector \( r \) or towards \( O' \))

(ii) If, only \( r \) varies and \( \theta \) constant it then resolves a rectilinear motion along a fixed direction \( \theta \)

\[ \theta = \text{constant} \]
\[ \theta = 0 \]
\[ v_\theta = 0 \]
\[ v_r = ra_\theta = 0 \]
\[ a_r = r \]

Example 7: The plane curvilinear motion of particle is defined in polar coordinates by \( r = t^3/3 + 2t \) and \( \theta = 0.3t^2 \). Find the magnitude of velocity, acceleration of path when \( t = 1 \) s.

Solution: Equations of motion are
\[ r = t^3/3 + 2t \quad \& \quad \theta = 0.3t^2 \]

Evaluating \( \theta \), \( d\theta/dt \). \( d^2\theta/dt^2 \)
\[ \theta = 0.3t^2 \]
\[ d\theta/dt = 2*0.3t \]
\[ d^2\theta/dt^2 = 0.6 \]

At \( t = 1 \) s
\[ \theta = 0.3 \text{ rad} \]
\[ d\theta/dt = 0.6 \text{ rad} \]
\[ d^2\theta/dt^2 = 0.6 \text{ rad/s}^2 \]
\[ r = 2.33m \text{ dr/dt} \]
\[ = 3m/s \text{ d}^2r/dt^2 \]
\[ = 2m/s^2 \]

Velocity
\[ V_r = dr/dt = 3m/s \]
\[ v_\theta = r\theta = 2.33*0.6 \]

**PROJECTILES**

Whenever a particle is projected upwards with some inclination to the horizontal (but not vertical), it travels in the air and traces a parabolic path and falls on the ground point (target) other than the point of projection. The particle itself is called projectile and the path traced by the projectile is called trajectory.
Terms used in projectile

1. Velocity of projection ($u$): It is the velocity with which projectile is projected in the upward direction with some inclination to the horizontal.

2. Angle of projection ($\alpha$): It is the angle with which the projectile is projected with respect to horizontal.

3. Time of flight ($T$): It is the total time required for the projectile to travel from the point of projection to the point of target.

4. Horizontal range ($R$): It is the horizontal distance between the point of projection and target point.

5. Vertical height ($h$): It is the vertical distance/height reached by the projectile from the point of projection.

Some relations

Time of flight:
Let $T$ be the time of flight. We know that the vertical ordinate at any point on the path of projectile after a time $T$ is given by

$$y = (u \sin \alpha)t - 0.5gt^2$$

When the projectile hits the ground, say at B: $y = 0$ at $t = T$
Above equation becomes

$$0 = (u \sin \alpha)t - 0.5gt^2$$
$$u \sin \alpha = 0.5gt$$
$$T = \frac{(2u \sin \alpha)}{g}$$

Horizontal range of the projectile:
During the time of flight, the horizontal component of velocity of projectile = $u \cos \alpha$
{Horizontal distance of the projectile} = $R$ = {Horizontal component of velocity of projection}

{Time of flight} = $u \cos \alpha \times T$
\[ R = (u \cos \alpha \times 2u \sin \alpha)/g = (u^2 \sin (2\alpha))/g \]

\[ \sin (2 \alpha) \text{ will be maximum only when } \sin 2 \alpha = 1 \]

\[ \sin 2 \alpha = \sin 90 \text{ or } \alpha = 45^\circ \]

Hence maximum horizontal range is given by

\[ R_{\text{max}} = (u^2 \sin 90)/g = u^2/g \]

Maximum height attained by the projectile: When the projectile reaches its maximum height, vertical component of velocity of projection becomes zero.

\[ V^2 - u^2 = 2gs \]

\[ 0 - u^2 \sin^2 \alpha = -2gh_{\text{max}} \]

\[ h_{\text{max}} = u^2 \sin^2 \alpha/2g \]

Time required to reach the maximum height is given by

\[ v = u + at \]

\[ 0 = u \sin \alpha - gt \]

\[ t = u \sin \alpha / g \]

**Motion of projectile:** Let a particle be projected upward from a point O at an angle \( \alpha \) with horizontal with an initial velocity of \( u \) m/s as shown in Figure 11.39. Now resolving this velocity into two components, we get

\[ u_x = u \sin \alpha \]

\[ u_y = u \cos \alpha \]

![Figure 5.15](image)

The vertical component of velocity is always affected by acceleration due to gravity. The particle will reach the maximum height when vertical component becomes zero. The horizontal component of velocity will remains constant since there is no effect of acceleration due to gravity. The combined effect of horizontal and vertical components of velocity will move the particle along some path in air and then fall on the ground other than the point of projection.
Equation for the path of projectile (Trajectory equation): Let a particle is projected at a certain angle from point O. The particle will move along a certain path OPA in the air and will fall down at A.

Let \( u \) = velocity of projection
\( \alpha = \) angle of projection

After \( t \) seconds, let a particle reach any point 'P' with \( x \) and \( y \) as coordinates as shown in Figure 5.16

We know that, horizontal component of velocity of projection = \( u \cos \alpha \)

Vertical component of velocity of projection = \( u \sin \alpha \)

Therefore,
\[
x = u \cos \alpha t
y = u \sin \alpha t - 0.5gt^2
\]

From Eq. (1)
\[
t = x/(u \cos \alpha)
\]

substitute in Eq. (2), we get
\[
y = u \sin \alpha [x/(u \cos \alpha)] - 0.5g[x/(ucos\alpha)]^2
y = x \tan \alpha - [gx^2/(2u^2 \cos^2 \alpha)]
\]

Figure 5.16

Example 8: A particle is projected at an angle of 60° with horizontal. The horizontal range of particle is 5 km. Find

(i) Velocity of projection (ii) Maximum height attained by the particle

Solution
Data given; \( R = 5 \text{ km} = 5000 \text{ m} \), \( g = 9.81 \text{ m/s}^2 \) and \( \alpha = 60° \)

To find: \( u \) and \( h_{\text{max}} \)

We know that
\[
R = (u^2 \sin 2\alpha)/g
\]

Substituting the known values in Eq. (1), we get
\[ u = 237.98 \text{ m/s} \]

Again, maximum height attained by the particle
\[ h_{\text{max}} = \frac{u^2 \sin \alpha}{2g} = 2164.9 \text{m} \]

**Motion of a body thrown horizontally from a certain height into air**

The figure shows a body thrown horizontally from certain height 'H' into air. At 'B' there is only horizontal component of velocity. As the body moves in the air towards the ground, the body has both horizontal and vertical components of velocity.

The horizontal component of velocity from B to A remains constant and will be equal to \( u \). But the vertical component of velocity in the downward direction will be subjected to gravitational force and hence will not be a constant.

Resultant velocity \( R = \sqrt{u^2 + v^2} \) \& \( \Theta = \tan^{-1}(v/u) \)

(i) Vertical downward distance travelled by the body is given by
\[ H = (\text{vertical component of velocity at B})t + 0.5gt^2 \]

(ii) The horizontal distance travelled by the body
\[ R = (\text{horizontal component of velocity at B})t \]
\[ R = ut \]

(iii) The vertical component of velocity at point A is obtained from the equation
\[ v = u + gt \]

or \[ v = gt \]

Resultant Velocity at A = \( R = \sqrt{u^2 + v^2} \)

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**Figure 5.17**
Example 9: An aircraft is moving horizontally at a speed of 108 km/h at an altitude of 1000 m towards a target on the ground releases a bomb which hits the target. Estimate the horizontal distance of aircraft from the target when it release a bomb. Calculate also the direction and velocity with which bomb hits the target.

Solution
Speed of aircraft = (108x100)/(60x30) = 30m/s
Horizontal velocity of bomb = u = 200 m/s
Height H= 1000m

Let t be the time required for the bomb to hit the target
We know that
\[ H = 0.5gt^2 \]
\[ 1000 = 0.5 \times 9.81 \times t^2 \]
or
\[ t = 14.278 \text{ s} \]

(i) Horizontal distance of aircraft from the target when it releases a bomb.
We know that
\[ R = u \times t = 30 \times 14.278 = 428.57 \text{ m} \]

(ii) Velocity with which bomb hits the target.
Vertical component of velocity = \( v = gt = 9.8 \times 14.278 = 139.9 \text{ m/s} \)
Horizontal component of velocity = \( u = 30 \text{ m/s} \)

Resultant velocity = \( R = \sqrt{u^2 + v^2} = 143.08 \text{ m/s} \)
Direction = \( \Theta = \tan^{-1} \left( \frac{v}{u} \right) = 77 \)